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**Abstract**

In this research we aim to study a day-ahead energy market from the point of view of a single producer participating with his rivals in energy biddings. The goal is to find the single producer’s best bidding strategy while the total cost of wholesale electricity supply for Independent System Operator (ISO) is minimized and all demand is met. The producer does not have full knowledge of market parameters. Hence, bidding prices of his rivals as well as energy demand are considered uncertain. A stochastic mixed integer bi-level optimization model is developed to address this problem. Discrete variables that are used in the second level to represent the commitment of the production units prohibit the application of traditional methodologies such as first order optimality conditions for model reformulation. Bi-level problems are NP-hard, however due to the special structure of our model we are able to develop an effective algorithm which could find the exact optimal solution for our stochastic bi-level model.

1. **Introduction**

The problem of electricity markets has led to market design and algorithmic issues addressed now for many years, which still provide with interesting research questions [1]. In this concept, there are various types of electricity markets. In regulated market all energy providing processes including pricing are governed by a regulatory or government body, with only the local utility able to sell directly to consumers. The utility or government set the prices for natural electricity supply, along with the associated transportation and distribution costs associated with those commodities. On the other hand, to establish a competitive electricity market, deregulation has taken place in many states and provinces throughout North America. It has allowed competitive energy suppliers, such to enter the markets and offer their energy supply products to consumers. Energy prices are not regulated in these areas and consumers are not forced to receive supply from their utility. In deregulated markets, consumers can choose their supplier. The marketing of these services is still regulated. Implementation of price deregulation requires open markets and transparent pricing. Moreover, majority of electricity markets designs establish either a wholesale or a retail electricity market that can operate in long-term and short-term horizons. At the day-ahead level of the wholesale electricity market, energy producers bid freely (usually subject to a cap) their energy production. An independent system operator (ISO) clears the market, allocating quantities to the participating producers, so as to minimize the total system bid-cost satisfying the demand for energy[[1](#_heading=h.3rdcrjn)]. Irrespective of the exact definition of deregulation it is clear that only part of the market is actually deregulated, while other parts are kept regulated.

In our work, we consider a day-ahead energy market consisting of multiple biding energy producers and study the system from the point of view of a single (individual) producer that participates in the market. We assume that this producer has partial knowledge of the demand for energy and the bids/costs of all the other producers, in the way that it can predict possible scenarios of energy demand and bid costs with a specific probability.

1. **Literature Review**

Many of the published works related to optimal bidding strategies has proposed forecasting methods for predicting the market’s clearing price (e.g. [[2](#_heading=h.26in1rg)]). Other works have used stochastic programming or bi-level optimization techniques in order to deal with market parameters uncertainties and finding optimal bidding strategies for a single producer. In these cases, a reformulation or a heuristic solution procedure has been proposed in order to solve the problem.

A bi-level optimization model has been developed by Weber and Overbye [[3](#_heading=h.lnxbz9)] in order to maximize welfare and an iterative search algorithm has been deployed to find its solution. Bi-level programming is proven to be NP-hard [[4](#_heading=h.35nkun2)]. A two-level stochastic optimization models for the same problem has been proposed by Gountis and Bakirtzis [[5](#_heading=h.1ksv4uv)] , and Fampa et al [[6](#_heading=h.44sinio)] and a heuristic procedure as well as a mixed integer reformulation was proposed to solve the problem. Some other researchers have developed bi-level optimization models and in order to reformulate and convert the problems into a mixed integer linear programs, has used the first order optimality conditions of the lower level problem [[7](#_heading=h.2jxsxqh)], [[8](#_heading=h.z337ya)], [[9](#_heading=h.3j2qqm3)], and [[10](#_heading=h.1y810tw)]. Kardakos et al [[11](#_heading=h.4i7ojhp)] has developed a two-stage stochastic bi-level optimization model and since the problem is very difficult to solve due to its non-linear a non-convex nature, a transformation of it as a mixed integer programming (MILP) was considered using previous studies’ approaches.

There also has been some works that has used other methods, such as a penalty interior point algorithm, sensitivity functions, particle swarm optimization, primal-dual interior point method, ε-constraint reduced feasible region method in order to solve the problem [[12](#_heading=h.2xcytpi)], [[13](#_heading=h.1ci93xb)], [[14](#_heading=h.3whwml4)], [[15](#_heading=h.2bn6wsx)], [[16](#_heading=h.qsh70q)].

Kozanidis et al [[1](#_heading=h.3rdcrjn)], has used a slightly different approach in that they have used binary variables to model the commitment of the electricity generation units as well as strictly positive lower bounds for the energy quantities of producers. As a result they could not use first order optimality conditions in order to simplify the model formulation. Instead, they have proposed an algorithmic methodology base on the theory of mixed integer parametric programming to solve the problem.

In this research, we have extended the work of Kozanidis et al [[1](#_heading=h.3rdcrjn)] in that we have considered uncertainy of market parameters such as demand load and the producer rivals’ offers. Hence the single producer that wants to maximize his profit does not have full knowledge of other producers’ bids and only can consider some possible scenarios with certain probabilities. We have formulated this problem as a stochastic mixed integer bi-level optimization model which aims to define the producer’s optimal energy unit price-bid. Then we try to develop a suitable algorithm to solve the problem by extending the proposed algorithm of Kozanidis et al [[1](#_heading=h.3rdcrjn)] and adopt it for our stochastic mixed integer bi-level optimization program.

The rest of this report is organized as follows. Section III briefly introduces bi-level programming as well as stochastic programing structure. In section IV problem assumptions are defined and the mathematical model is developed. Section VI discusses challenges that we might face to complete our work and concluding remarks are mentioned.

1. **A Brief Review of Bi-level Programing and Stochastic Programing**

Bi-level optimization is a special kind of [optimization](https://en.wikipedia.org/wiki/Optimization) where one problem is embedded (nested) within another. The outer optimization task is commonly referred to as the upper-level optimization task, and the inner optimization task is commonly referred to as the lower-level optimization task. A general formulation of bi-level optimization problem can be written as follows:

|  | |  | | (1) |
| --- | --- | --- | --- | --- |
| S.t: , |  | | | (2) |
|  | | |  | (3) |

In the above formulation, represents the upper-level objective function and represents the lower-level objective function. Similarly represents the upper-level decision vector and represents the lower-level decision vector. and represent the inequality constraint functions at the upper and lower levels respectively. In general each of the mentioned levels can be considered as an agent who tries to optimize its respective objective functions over a jointly dependent set [[17](#_heading=h.3as4poj)].

On the other hand, for the problems that involve uncertainty, stochastic programming is used for [modeling](https://en.wikipedia.org/wiki/Mathematical_model) the [optimization](https://en.wikipedia.org/wiki/Optimization_(mathematics)) problem. In stochastic optimization in contrary of deterministic optimization, parameters are unknown but the [probability distributions](https://en.wikipedia.org/wiki/Probability_distributions) governing the parameters are known or can be estimated. Stochastic optimization can be addressed with chance constrained technique, two-stage stochastic programing approach, etc. In two-stage stochastic programming model, the first stage decisions, which are called design variables, are made before observing the random outcome at the second-stage. The second stage decision corresponds to ‘recourse’, once all randomness has been removed.

|  | |  | (4) |
| --- | --- | --- | --- |
| S.t: , |  | | (5) |
|  |  | | (6) |
|  |  | | (7) |

In such formulation corresponds to recourse, is the first-stage decision variable vector, is the second-stage decision variable vector, and is the objective function of the second-stage problem. The goal is to find some policy that is feasible for all (or almost all) the recourses and maximizes the expectation of some function of the decisions [variables](https://en.wikipedia.org/wiki/Random_variable).

1. **Bi-level Programing and Stochastic Programing Application**

Bi-level programming, has lots of application in different fields, especially in pricing, which considered the leadership part as decision maker in upper level and customer behavior as followers in lower level in passenger transport pricing problem. Applying bi-level programming could make a balance in agencies and travelers expectations. [[18](#_heading=h.1pxezwc)]

Bi-level programming model, also formed in Transition Networks (TN), to conduct stochastic programming in two level, first on generating the uncertainties and second in optimizing stochastic model. In this model, the first level is an MIP and second is a linear model and due to minimum load shedding as the objective function of both level model, the intervals would be ended when there is no more improvement in objective functions. [[19](#_heading=h.49x2ik5)]

According to stochastic nature of wind energy, in power system, stochastic programming mostly used in power system problem with wind farm.

Optimal bidding strategy problem is one of the effective problems for applying bi-level stochastic programming, regarding uncertainties of wind and hydro energy as the resource of generating power. This problem has been solved by merging upper level (MIP) and lower level (LP) by adding Karush-Kuhn-Tucker (KKT) condition of LP problem to upper level. [[20](#_heading=h.2p2csry)]

Same application in the optimal bidding strategy of Commercial Virtual Power Plant (CVPP) in Day Ahead (DA) electricity market. Here, the problem is modeled as a three-stage stochastic bi-level programming and solved by same approach as previous ones, adding KKT optimality conditions considering strong duality theorem to achieve optimal solution. So, it has been solved as mixed integer problem. Uncertainties in this model, are market price and resource of power generation. [[21](#_heading=h.147n2zr)]

Applying bi-level stochastic modeling, in upper level, maximizing the profit for Wind Power Producers (WPPs) is targeted and in lower level on side of Independent System Operator (ISO) the market clearing problem is modeled. Also the equilibrium constraints took into account in the stochastic mathematical problem. The same as previous ones, the uncertainties of wind power generation and balanced market price has been taken into account. In this order, scenarios generated by finding possible outcomes of random input based on corresponding occurrence probability. [[22](#_heading=h.3o7alnk)]

This problem studied by same structure in the upper and lower level definition but different uncertainties e.g. uncertainties on DA system load and the producer rival’s offer. Considering the lower level, linear, first order KKT condition of it added to upper level and based on KKT optimality condition and strong duality, the problem solved. Also, the risk on producer profit variability, Conditional Value-at-Risk (CVAR) metric is incorporated. So, the producer could decide the desired risk wants to take.[[11](#_heading=h.4i7ojhp)]

Here for our problem, we use bi-level programing in order to maximize the profit of the single producer in the upper level while simultaneously minimizing the total cost of providing energy demand by all producers in the lower level. Moreover, the uncertainty of demand and bidding price of the producers’ rivals are addressed by considering scenarios with a certain probability and incorporating two-stage stochastic programing into the bi-level approach. In the next section, model formulation is described in detail.

1. **Mathematical Model Formulation**

The problem on the hand addresses a day-ahead electricity market in which producers submit energy offers to ISO, and based on the bids ISO clear the market and decide which generation units should commit and how much power they should provide. The problem is considered from the point of view of a single individual producer that does not have full knowledge of other bidder prices and only can estimate some possible prices that the bidders might submit to ISO. Moreover, we consider the energy demand of consumers to be uncertain. As a result, a stochastic mixed integer bi-level optimization model is developed that addresses uncertainty of demand as well as rivals’ bidding prices. In the first level of this model we try to maximize the single producers profit while in the second level we try to minimize total cost of the system,

For the mathematical formulation, we the following notation is used:

Sets:

|  | Generating units except generating unit 1, indexed by g |
| --- | --- |
|  | Scenarios of demand and rivals’ prices, indexed by s |

Decision Variables:

|  | Energy price-bid of individual producer, i.e., of unit 1 |
| --- | --- |
|  | Energy quantity of unit 1 in scenario s |
|  | Binary variable that takes the value 1 if unit g produces a positive energy quantity in scenario s, and 0 otherwise |
|  | Energy quantity of unit g in scenario s |
|  | Binary variable that takes the value 1 if unit g produces a positive energy quantity in scenario s, and 0 otherwise |
|  | Shadow price of the market clearing constraint in scenario s, that ensures satisfaction of the demand for energy (market clearing price) |

Parameters:

|  | Energy price-bid of unit g in scenario s |
| --- | --- |
|  | Technical maximum of unit g |
|  | Technical minimum of unit g |
|  | Price cap for the energy offer |
|  | Variable cost of the individual producer, i.e. of unit 1 |
|  | Start-up cost of unit g |
|  | Demand for energy in scenario s |
|  | Probability of occurrence of scenario s |

The mathematical model is formulated as follows:

|  | | | |  | | | | | | | (8) |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| S.t: | , | | |  | | | | | | | (9) |
|  | | | |  | | | | | | |  |
|  | | | | | | |  | | | | (10) |
| S.t: |  |  |  | | | | |  | | | (11) |
|  | | | | |  | | | |  | | (12) |
|  | | | | | |  | | | |  | (13) |

The objective function (8) maximizes the individual producer’s profit. This profit depends on expected value of the market clearing price, , which is the shadow price of the market clearing constraint (11) in scenario s, that ensures satisfaction of the demand.

Constraint (9) imposes a lower and an upper bound on the price-bid of the individual producer. The lower level problem is defined by (10)-(12). The objective function (10) minimizes the total bid-cost for providing energy. Constraint (11) clears the market by ensuring that the demand for energy is satisfied. Constraint (12) and ensures that the technical minimum and the technical maximum of each generation unit are not violated. Finally, constraint (13) impose integrality and non-negativity restrictions, respectively to decision variables.

Although many efficient algorithms and software packages have been designed and developed for general or structured single-level MIP, computing bi-level MIP remains challenging. Because of the special structure of model that we are addressing, we are able to provide exact algorithm that offers the optimal solution in a reasonable time.

1. **Solution Methodology**

Before we proceed with the solution methodology, first we define some notations and some preliminaries. We denote the optimum objective value of second level problem for a fixed value by and the corresponding solution by in which is the vector of start-up decision variables and is the vector of production level decision variable for all scenarios).

For each interval where we define the line as a parametric function with single parameter.

(15)

in which:

And

In which are the optimum solution of the second level problem for a fixed value of which yields the objective value of. is piecewise-linear and concave in its finite domain [[23](#_heading=h.23ckvvd)]. Since is just used in the objective function changing it won’t change the feasible region. Also is positive, so it is a non-decreasing function of. Because of these properties

In this algorithm we use Geoffrion and Nauss’s method [[24](#_heading=h.ihv636)] which is based on the following proposition. It assures the convergence of the algorithm.

Proposition:

Kozanidis et al [[1](#_heading=h.3rdcrjn)] used Geoffrion and Nauss’s [[24](#_heading=h.ihv636)] method to solve the non-stochastic version of the problem. We adapt the algorithm to solve the stochastic model.

For simplicity in our algorithm we rewrite (15) as:

where in can vary in interval which .

Also () is the optimal solution of second level problem in which is fixed at and likewise we obtain when is fixed at.

Also is the line in which and are obtained when the (*u,Q*) is fixed at and can moves in from to . Likewise is the line in which and are obtained when the (*u,Q*) is fixed at and can move in from towards .



b

b

)

)

)

)

*Figure 1 graph of and*

In this approach shadow price plays a critical role. O'Neill et al [[25](#_heading=h.32hioqz)] used the following method for mixed integer programming (MIP). They first solved the MIP problem and then fixed the integer values to their optimal and solved the continuous LP to get shadow prices for demand constraint. Here we need to calculate shadow price for each demand constraint corresponding to each scenario.

1. **Solution algorithm**

The output of this algorithm is several intervals and the optimum value corresponding to that interval. Given, one can calculate for that interval as well. Also for each interval denotes the shadow price corresponding the demand constraint associated with that scenario.

1. Initialize and
2. Fix at and solve the second level problem to obtain the optimal solution (). Do the same for to obtain the optimal solution (). If go to 3 otherwise go to 4.
3. The interval is processed and the optimal solution of the lower level problem is the same for any feasible value of. Do the following and then go to step 7.
   1. If then
   2. If then.
4. Find the intersection for lines and and find the. If go to 5. Otherwise go to 6.
5. The interval is processed and we have two sub intervals. Do the following for those intervals and go to 7.
   1. The interval. () is the optimal lower level problem solution for each in. For each scenario If then otherwise.
   2. The interval. () is the optimal lower level problem solution for each in. For each scenario If then otherwise.
6. Do the following subs steps and go to step 7.
   1. Set and and go to step 2.
   2. Set and and go to step 2
7. If there exist unprocessed interval we go to step 1 and set equal to that unprocessed interval. Otherwise go to step 8.
8. Let I be the set of intervals obtained in the previous steps and be the best value of for that interval for scenario s. Then the optimal to the main problem will be obtained as following:

The above algorithm divides the interval to sub intervals for which the optimal value for lower level problem within that subinterval can be determined (step 4). Since the feasible region is bounded, there are a finite number of such subintervals which gives a finite number of segments to the piecewise linear optimal value function [[24](#_heading=h.ihv636)] and hence we will have a bounded objective value which can optimized in finite steps. For each subinterval the price bid for which the lower level has the optimum value ( and the corresponding solution is determined (steps 2 and 5). If the optimum for that subinterval cannot be determined, then we divide the subinterval to two subintervals and we continue dividing until we find a subinterval for which the optimal can be calculated (steps 4, 6). When we found appropriate subintervals and determined the optimum solution and, we compare to shadow price of each demand constraint corresponding to scenarios to determine (steps 3 and 5). It will give a more realistic solution than comparing the with the expected value of shadow prices.

The price that will be paid to the individual producer will be the shadow price of the demand constraint in the sub problem. However in some cases the individual may be able to determine the shadow price. It is that . It means that the individual producer determines the system marginal price for that scenario. For this subinterval we set the price bid as high as possible () because while it maintains the optimality of the lower level problem, it creates the more profit for the individual producer.

The other case happens when which shows that the system marginal price is determined by another producer and the producer must adopt it as his optimizer bid price.

Note that in either cases the quantity that producer can provide is determined by the lower level problem. By comparing the maximum profit that the individual producer can realize in any subinterval we can easily identify the optimal value of that results in his maximum profit (step 8).

1. **Numerical results**

In this section, we are applying the above algorithm for the case of 3 producers and 2 scenarios.

|  | | Producer1 | Producer2 | Producer3 | Demand |
| --- | --- | --- | --- | --- | --- |
| Price | Scenario 1 | 10 | 11 | 10.5 | 270 |
| Scenario 2 | 10 | 11.5 | 12 | 300 |
|  | | 50 | 70 | 90 |  |
|  | | 100 | 150 | 190 |  |
| Startup cost | | 25 | 25 | 30 |  |

c1=7 and . The probability of scenario 1 is 0.3 and the probability for scenario 2 is 0.3.

We coded the algorithm in C++ using IBM CPLEX solver. The result is as table 2

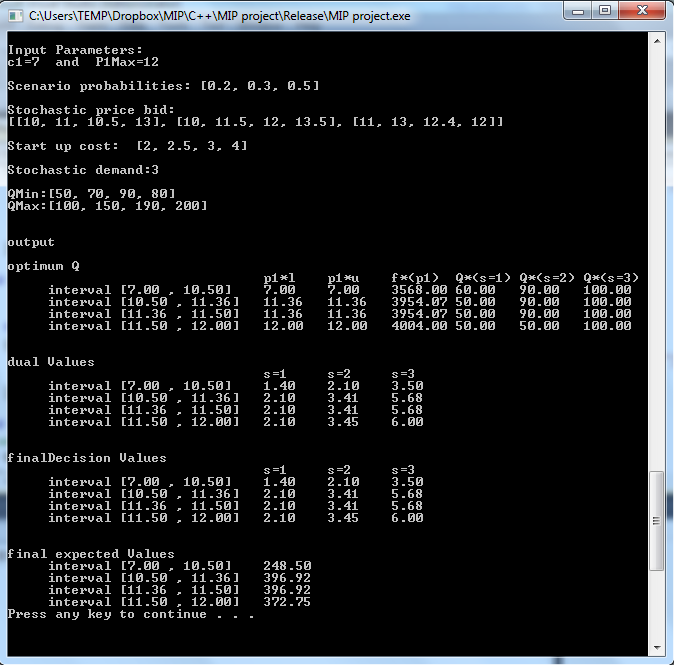
|  | Lower Level Optimal Solution for producer 1 | | system marginal price | |  |  | |
| --- | --- | --- | --- | --- | --- | --- | --- |
| subinterval | Scenario 1 | scenario2 | Scenario 1 | scenario2 | Scenario 1 | scenario2 |
| [7 , 10.5] | 100 | 100 | 3.15 | 8.05 | 7 | 3.15 | 8.05 |
| [10.5 , 11.1512] | 50 | 100 | 3.15 | 8.05 | 11.1512 | 3.15 | 8.05 |
| [11.1512 , 11.5] | 50 | 100 | 3.15 | 8.05 | 11.1512 | 3.15 | 8.05 |
| [11.5 , 12] | 50 | 60 | 3.15 | 8.4 | 12 | 3.15 | 8.4 |

Hence we can calculate the step 8 for each subinterval.

| subinterval |  |
| --- | --- |
| [7,10.5] | 658 |
| [10.5,11.1512] | 610.75 |
| [11.1512,11.5] | 610.75 |
| [11.5,12] | 400.05 |

And the value occurs in the first interval with expected value of is 658.

Example 2: The picture below is the output of a problem with 4 producers and 3 scenarios. The optimum happens on two middle intervals which have the same values for optimal and .



1. **Conclusion**

The proposed model is a realistic extension of Kozanidis et al [[1](#_heading=h.3rdcrjn)] model to incorporate uncertainty in demand and bidding price of the energy markets participants. Since this problem is considered from the view of an individual producer, a stochastic mixed integer bi-level optimization model is designed. The first level of the problem tries to maximize the profit of the single producer and in the second, ISO clears the market and decides which units should commit and how much energy should be dispatched. Our next step is to develop a suitable algorithm to solve the problem. The biggest challenge is figuring how to come over the second stage which is not an LP because of discrete variables; we cannot use traditional methodologies, such as first order optimality conditions for model reformulation. As a result, we need to develop an algorithm for our proposed model based on the proposed algorithm of [[1](#_heading=h.3rdcrjn)] as well as theory of mixed integer parametric programming. One of the issues is that objective function has more terms and becomes much more complicated because of the expected values of functions of our scenarios. Another concern is that in the proposed algorithm of [[1](#_heading=h.3rdcrjn)], there is a single shadow price that associates with the market clearing constraint. But, here, since we have incorporated uncertainty in our model, there are several shadow prices due to several market clearing constraints (for each scenario there is a market clearing constraint). The difficulty for this portion is that if we use L-Shaped algorithm for solving the lower level problem, we cannot easily calculate shadow prices. So, even if LP relaxation applied in the second level and combine both levels by writing KKT optimality condition of the second level model and linearize respected constraints, still the objective function has been remained non-linear. So, finding upper bound based on LP relaxation was not possible. Therefore, we proposed an algorithm which is modified algorithm of [1] which is updated based on stochastic nature of our model. According to our solution approach we got exact optimal solution as provided before.

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